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球面正压粘性大气的运动特征

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提　要

通过对球面正压粘性大气非线性扰动的研究表明:由于波动之间的弱非线性相互作用,球面小扰动的波包演变是由一种较为简单的方程形式控制,当此粘性力消失时,该方程为非线性 Schrödinger 方程。研究结果还表明:非线性 Schrödinger 孤立波的形成,有两个因子非常重要,一是运动的球面性,另一个则是基本风场的经向切变。前者反映了球面运动的特殊性。本文还从另一个角度证明了粘性力的作用对波动起着加强能量的耗散作用,不利于这种几率波(Schrödinger 孤立波)的形成。

一、引　言

研究大气大尺度系统的运动时,必须考虑地球的球面性特点,即采用球坐标系。大气中阻塞形势的时、空尺度都是比较大的,任何平面近似(如 f 平面近似或 β 平面近似)下的坐标都会曲解它的运动特点。阻塞形势对中纬度地区的天气过程起着主导作用,研究它的结构及运动特点有利于我们认识中纬度地区的演变。阻塞高压一般形成于中高纬地区,有时它的生命史可长达一个半月之久。如此长时间地维持足以说明:阻塞高压的能量频散是很弱的。叶笃正指出:Rossby 波的能量频散是与 Rossby 参数 β 成正比的[1]。在较高纬度地区的天气系统,其能量频散是较小的。在较低纬度地区,科氏参数 f 较小,所以,侵入较低纬度地区的天气系统容易破碎而难以维持。一般来说,阻塞高压往往是和基本风场的经向切变相联系的。

本文研究表明大气中基本风场的经向切变是阻塞高压形成的一个很关键的因子。

球面正压粘性大气的扰动演变,其扰动波幅仍处于相对小的阶段,波动之间的非线性相互作用是一种弱非线性过程,这种扰动的波包络即为球面 Rossby 波群的调制结果。本文所讨论的弱非线性相互作用是在扰动幅度较小的前提下进行的。实际上,中高纬地区的一些扰动已经是强非线性问题,尤其是在阻塞形势中这种波动幅度比较大的阶段更是如此,但作为定性的认识,这样的假设是可以的。

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二、球面非线性扰动的波包

根据文献[2]可得球面正压无辐散粘性大气的涡度方程为：

$$(\frac{\partial}{\partial t}+v_\theta\frac{\partial}{a\partial\theta}+v_\lambda\cdot\frac{\partial}{a\sin\theta\partial\lambda})(2\Omega\cos\theta+a^{-2}\Delta\psi)=-Ka^{-2}\Delta\psi \tag{1}$$

式(1)中略去了与$\frac{\mathrm{ctg}\theta}{a^2}\frac{\partial\psi}{\partial\theta}$有关的项，并且认为大气运动的粘性力$(D_\theta、D_\lambda)=(-Kv_\theta、-Kv_\lambda)$[3]。$K$为粘性系数，$\psi$为流函数，$a$为地球半径，且

$$v_\theta=-\frac{\partial\psi}{a\sin\theta\partial\lambda}\qquad v_\lambda=\frac{\partial\psi}{a\partial\theta} \tag{2}$$

取$\varphi=90°-\theta$代表相应于θ(余纬)的纬度，则式(1)变为：

$$(\frac{\partial}{\partial t}+\frac{\partial\psi}{a\cos\varphi\partial\lambda}\cdot\frac{\partial}{a\partial\varphi}-\frac{\partial\psi}{a\partial\varphi}\frac{\partial}{a\cos\varphi\partial\lambda})(2\Omega\sin\varphi+a^{-2}\Delta\psi)=-Ka^{-2}\Delta\psi \tag{3}$$

为了研究阻塞形势，将流场分为基本流场和因扰动引起的扰动流场，即令：

$$u=-\frac{\partial\psi}{a\partial\varphi}=\bar{u}+u' \tag{4}$$

$$v=\frac{\partial\psi}{a\cos\varphi\partial\lambda}=\bar{v}+v' \tag{5}$$

其中，“—”量表示平均流场(基本流场)，“′”量表示由扰动运动引起的扰动流场。一般认为基本风场的经向分量很弱，即：

$$\bar{v}=0 \tag{6}$$

故有

$$\psi=\bar{\psi}(\varphi)+\psi'(\lambda,\varphi,t) \tag{7}$$

即认为：基本流场在经向是有变化的(这是基于考虑阻塞形势一般对应有较强的基本风场的经向切变)。将(7)式代入(3)式，并认为基本流场亦满足(3)式，立即可得到扰动方程：

$$\begin{aligned}(\frac{\partial}{\partial t}-\frac{\partial\bar{\psi}}{a\partial\varphi}\cdot\frac{\partial}{a\cos\varphi\partial\lambda})(a^{-2}\Delta\psi')+\frac{\partial\psi'}{a\cos\varphi\partial\lambda}\cdot\frac{\partial}{a\partial\varphi}(2\Omega\sin\varphi+a^{-2}\Delta\bar{\psi})\\+\frac{\partial\psi'}{a\cos\varphi\partial\lambda}\cdot\frac{\partial}{a\partial\varphi}(a^{-2}\Delta\psi')-\frac{\partial\psi'}{a\partial\varphi}\cdot\frac{\partial}{a\cos\varphi\partial\lambda}(a^{-2}\Delta\psi')=Ka^{-2}\Delta\psi\end{aligned} \tag{8}$$

由于所讨论的阻塞形式不是出现在边界层，因此可以认为粘性力的作用相对来说比左边各项的量级要小，若取

$$K=\delta\varepsilon \tag{9}$$

δ为放大的粘性力系数，这种放大的粘性力应与左边各项具有相当的量级，而$\varepsilon=\frac{K}{\delta}$则必然是小于1的正数。取$\varepsilon\ll1$(因为考虑的粘性力较小)，将(9)代入(8)式，然后采用多重尺度方法来研究(8)式的非线性运动特点。将ψ'按小参数ε作幂级数展开：

$$\psi'=\varepsilon\psi_0+\varepsilon^2\psi_1+\varepsilon^3\psi_2+\cdots \tag{10}$$

(9)式的选取有一定的限制：如果粘性力更小，可取$K=\delta\varepsilon^2$；或粘性力更大一点，则取$K=\delta\sqrt{\varepsilon}$等。所以，适当地选取小参数$\varepsilon$的量级，方程的特性也就被决定了，即扰动的特点也就决定了。

又据$\psi=\psi_0+\psi'$及(10)式可得$\varepsilon\sim0(\psi_0+\psi')$，即相当于扰动流场的幅度与基本运动流

场幅度的对比。由此亦可大致确定 ε 的大小。

又由于 Rossby 波是色散波,而扰动又是 Rossby 波群调制而成的,因而可以认为扰动具有多重尺度特点,故可以引入以下的缓变时、空尺度:

$$\begin{cases}\tau=\varepsilon t & T=\varepsilon^2 t\\ \lambda_1=\varepsilon\lambda & \lambda_2=\varepsilon^2\lambda\end{cases}\tag{11}$$

这里在经向没有引入缓变坐标,只有快变量 φ,这是基于考虑阻塞形势在经向具有较为复杂的结构。相应的坐标微分形式变为:

$$\begin{cases}\dfrac{\partial}{\partial t}\to\dfrac{\partial}{\partial t}+\varepsilon\dfrac{\partial}{\partial\tau}+\varepsilon^2\dfrac{\partial}{\partial T}\\ \dfrac{\partial}{\partial\varphi}\to\dfrac{\partial}{\partial\varphi}\\ \dfrac{\partial}{\partial\lambda}\to\dfrac{\partial}{\partial\lambda}+\varepsilon\dfrac{\partial}{\partial\lambda_1}+\varepsilon^2\dfrac{\partial}{\partial\lambda_2}\end{cases}\tag{12}$$

将(10)、(11)、(12)三式代入(8)式,得到:

$$a^{-2}[\frac{\partial}{\partial t}+\varepsilon\frac{\partial}{\partial\tau}+\varepsilon^2\frac{\partial}{\partial T}+\frac{\bar{u}}{a\cos\varphi}(\frac{\partial}{\partial\lambda}+\varepsilon\frac{\partial}{\partial\lambda_1}+\varepsilon^2\frac{\partial}{\partial\lambda_2})]\cdot[\frac{\partial^2}{\partial\varphi^2}-\mathrm{tg}\varphi\frac{\partial}{\partial\varphi}+\frac{1}{\cos^2\varphi}(\frac{\partial^2}{\partial\lambda^2}+2\varepsilon\frac{\partial^2}{\partial\lambda\partial\lambda_1}+2\varepsilon^2\frac{\partial^2}{\partial\lambda\partial\lambda_2}+\varepsilon^2\frac{\partial^2}{\partial\lambda_1^2})]\cdot(\varepsilon\psi_0+\varepsilon^2\psi_1+\varepsilon^3\psi_2)+\frac{1}{a^2\cos\varphi}\frac{\partial}{\partial\varphi}[(2\Omega\sin\varphi+a^{-2}\Delta_1\bar{\psi})](\frac{\partial}{\partial\lambda}+\varepsilon\frac{\partial}{\partial\lambda_1}+\varepsilon^2\frac{\partial}{\partial\lambda_2})(\varepsilon\psi_0+\varepsilon^2\psi_1+\varepsilon^3\psi_2)+\frac{1}{a^4\cos^2\varphi}[(\frac{\partial}{\partial\lambda}+\varepsilon\frac{\partial}{\partial\lambda_1}+\varepsilon^2\frac{\partial}{\partial\lambda_2})\cdot(\varepsilon\psi_0+\varepsilon^2\psi_1+\varepsilon^3\psi_2)\frac{\partial}{\partial\varphi}(\frac{\partial^2}{\partial\varphi^2}-\mathrm{tg}\varphi\frac{\partial}{\partial\varphi}+\frac{1}{\cos^2\varphi}(\frac{\partial^2}{\partial\lambda^2}+2\varepsilon\frac{\partial^2}{\partial\lambda\partial\lambda_1}+2\varepsilon^2\frac{\partial^2}{\partial\lambda\partial\lambda_2}+\varepsilon^2\frac{\partial^2}{\partial\lambda_1^2}))(\varepsilon\psi_0+\varepsilon^2\psi_1+\varepsilon^3\psi_2)-\frac{\partial}{\partial\varphi}(\varepsilon\psi_0+\varepsilon^2\psi_1+\varepsilon^3\psi_2)(\frac{\partial}{\partial\lambda}+\varepsilon\frac{\partial}{\partial\lambda_1}+\varepsilon^2\frac{\partial}{\partial\lambda_2})(\frac{\partial^2}{\partial\varphi^2}-\mathrm{tg}\varphi\frac{\partial}{\partial\varphi}+\frac{1}{\cos^2\varphi}(\frac{\partial^2}{\partial\lambda^2}+2\varepsilon\frac{\partial^2}{\partial\lambda\partial\lambda_1}+2\varepsilon^2\frac{\partial^2}{\partial\lambda\partial\lambda_2}+\varepsilon^2\frac{\partial^2}{\partial\lambda_1^2}))(\varepsilon\psi_0+\varepsilon^2\psi_1+\varepsilon^3\psi_2)]=-\delta\varepsilon a^{-2}[\frac{\partial^2}{\partial\varphi^2}-\mathrm{tg}\varphi\frac{\partial}{\partial\varphi}+\frac{1}{\cos^2\varphi}(\frac{\partial^2}{\partial\lambda^2}+2\varepsilon\frac{\partial^2}{\partial\lambda\partial\lambda_1}+2\varepsilon^2\frac{\partial^2}{\partial\lambda\partial\lambda_2}+\varepsilon^2\frac{\partial^2}{\partial\lambda_1^2})](\varepsilon\psi_0+\varepsilon^2\psi_1+\varepsilon^3\psi_2)\tag{13}$$

按照多重尺度分析原则[4],因而有以下各阶近似的等式。

$0(\varepsilon^1)$:

$$a^{-2}(\frac{\partial}{\partial t}+\frac{\bar{u}}{a\cos\varphi}\frac{\partial}{\partial\lambda})(\frac{\partial^2}{\partial\varphi^2}-\mathrm{tg}\varphi\frac{\partial}{\partial\varphi}+\frac{1}{\cos^2\varphi}\frac{\partial^2}{\partial\lambda^2})\psi_0+[\frac{1}{a^2\cos\varphi}\frac{\partial}{\partial\varphi}(2\Omega\sin\varphi+a^{-2}\Delta_1\bar{\psi})]\frac{\partial\psi_0}{\partial\lambda}=0\tag{14}$$

$0(\varepsilon^2)$:

$$a^{-2}(\frac{\partial}{\partial t}+\frac{\bar{u}}{a\cos\varphi}\frac{\partial}{\partial\lambda})(\frac{\partial^2}{\partial\varphi^2}-\mathrm{tg}\varphi\frac{\partial}{\partial\varphi}+\frac{1}{\cos^2\varphi}\frac{\partial^2}{\partial\lambda^2})\psi_1+a^{-2}(\frac{\partial}{\partial t}+\bar{u}\frac{\partial}{a\cos\varphi\partial\lambda})(\frac{2}{\cos^2\varphi}\cdot\frac{\partial^2\psi_0}{\partial\lambda\partial\lambda_1})+a^{-2}(\frac{\partial}{\partial\tau}+\frac{\bar{u}}{a\cos\varphi}\frac{\partial}{\partial\lambda_1})\cdot(\frac{\partial^2}{\partial\varphi^2}-\mathrm{tg}\varphi\frac{\partial}{\partial\varphi}+\frac{1}{\cos^2\varphi}\frac{\partial^2}{\partial\lambda^2})\psi_0+[\frac{1}{a^2\cos\varphi}\frac{\partial}{\partial\varphi}(2\Omega\sin\varphi+a^{-2}\Delta_1\bar{\psi})]\cdot(\frac{\partial\psi_0}{\partial\lambda}+\frac{\partial\psi_0}{\partial\lambda_1})+\frac{1}{a^4\cos\varphi}[\frac{\partial\psi_0}{\partial\lambda}\frac{\partial}{\partial\varphi}(\frac{\partial^2}{\partial\varphi^2}-\mathrm{tg}\varphi\frac{\partial}{\partial\varphi}+\frac{1}{\cos^2\varphi}\frac{\partial^2}{\partial\lambda^2})\psi_0-\frac{\partial\psi_0}{\partial\lambda}\frac{\partial}{\partial\varphi}(\frac{\partial^2}{\partial\varphi^2}-\mathrm{tg}\varphi\frac{\partial}{\partial\varphi}+\frac{1}{\cos^2\varphi}\frac{\partial^2}{\partial\lambda^2})\psi_0]=-\delta(\frac{\partial^2}{\partial\varphi^2}-\mathrm{tg}\varphi\frac{\partial}{\partial\varphi}+\frac{1}{\cos^2}\frac{\partial^2}{\partial\lambda^2})\psi_0\tag{15}$$

$0(\varepsilon^3)$:

$$a^{-2}[(\frac{\partial}{\partial t}+\frac{\bar{u}}{a\cos\varphi}\frac{\partial}{\partial\lambda})(\frac{\partial^2}{\partial\varphi^2}-\mathrm{tg}\varphi\frac{\partial}{\partial\varphi}+\frac{1}{\cos^2\varphi}\frac{\partial^2}{\partial\lambda^2})\psi_2+(\frac{\partial}{\partial t}+\frac{\bar{u}}{a\cos\varphi}\frac{\partial}{\partial\lambda})(\frac{2}{\cos^2\varphi}\frac{\partial^2\psi_1}{\partial\lambda\partial\lambda_1})+(\frac{\partial}{\partial t}+$$

$$\frac{\bar{u}}{a\cos\varphi}\frac{\partial}{\partial\lambda})(\frac{2}{\cos^2\varphi}\frac{\partial^2\psi_0}{\partial\lambda\partial\lambda_2})+(\frac{\partial}{\partial\tau}+\frac{\bar{u}}{a\cos\varphi}\frac{\partial}{\partial\lambda_1})(\frac{\partial^2}{\partial\varphi^2}-\mathrm{tg}\varphi\frac{\partial}{\partial\varphi}+\frac{1}{\cos^2\varphi}\frac{\partial^2}{\partial\lambda^2})\psi_1+(\frac{\partial}{\partial\tau}+\frac{\bar{u}}{a\cos\varphi}\frac{\partial}{\partial\lambda_1})\cdot$$

$$(\frac{2}{\cos^2\varphi}\frac{\partial^2\psi_0}{\partial\lambda\partial\lambda_1})+(\frac{\partial}{\partial T}+\frac{\bar{u}}{a\cos\varphi}\frac{\partial}{\partial\lambda_2})(\frac{\partial^2}{\partial\varphi^2}-\mathrm{tg}\varphi\frac{\partial}{\partial\varphi}+\frac{1}{\cos^2\varphi}\frac{\partial^2}{\partial\lambda^2})\psi_0]+[\frac{1}{a^2\cos\varphi}\frac{\partial}{\partial\varphi}(2\Omega\sin\varphi+a^{-2}\Delta_1\bar{\psi})]$$

$$(\frac{\partial\psi_0}{\partial\lambda_2}+\frac{\partial\psi_1}{\partial\lambda_1}+\frac{\partial\psi_2}{\partial\lambda})+\frac{1}{a^4\cos\varphi}\{\frac{\partial\psi_0}{\partial\lambda}\frac{\partial}{\partial\varphi}(\frac{\partial^2\psi_1}{\partial\varphi^2}-\mathrm{tg}\varphi\frac{\partial\psi_1}{\partial\varphi}+\frac{1}{\cos^2\psi}\frac{\partial^2\psi_1}{\partial\lambda^2}+\frac{2}{\cos^2\varphi}\frac{\partial^2\psi_0}{\partial\lambda\partial\lambda_1})+(\frac{\partial\psi_1}{\partial\lambda}+\frac{\partial\psi_0}{\partial\lambda_1})\frac{\partial}{\partial\varphi}$$

$$(\frac{\partial^2}{\partial\varphi^2}-\mathrm{tg}\varphi\frac{\partial}{\partial\varphi}+\frac{1}{\cos^2\varphi}\frac{\partial^2}{\partial\lambda^2})\psi_0-[\frac{\partial\psi_0}{\partial\lambda}\frac{\partial}{\partial\lambda}(\frac{\partial^2}{\partial\varphi^2}-\mathrm{tg}\varphi\frac{\partial}{\partial\varphi}+\frac{1}{\cos^2\varphi}\frac{\partial^2}{\partial\lambda^2})\psi_1+\frac{\partial\psi_0}{\partial\varphi}\cdot\frac{2}{\cos^2\varphi}\cdot\frac{\partial^3\psi_0}{\partial\lambda_1\partial\lambda^2}+\frac{\partial\psi_0}{\partial\varphi}$$

$$\frac{\partial}{\partial\lambda_1}(\frac{\partial^2}{\partial\varphi^2}-\mathrm{tg}\varphi\frac{\partial}{\partial\varphi}+\frac{1}{\cos^2\varphi}\frac{\partial^2}{\partial\lambda^2})\psi_0+\frac{\partial\psi_1}{\partial\varphi}\frac{\partial}{\partial\lambda}(\frac{\partial^2}{\partial\varphi^2}-\mathrm{tg}\varphi\frac{\partial}{\partial\varphi}+\frac{1}{\cos^2\varphi}\frac{\partial^2}{\partial\lambda^2})\psi_0+\frac{\partial\psi_0}{\partial\varphi}\cdot\frac{2}{\cos^2\varphi}\frac{\partial^3\psi_0}{\partial\lambda_1\partial\lambda^2}]\}=-$$

$$a^{-2}\delta(\frac{\partial^2}{\partial\varphi^2}-\mathrm{tg}\varphi\frac{\partial}{\partial\varphi}+\frac{1}{\cos^2\varphi}\cdot\frac{\partial^2}{\partial\lambda^2})\psi_1-\delta\frac{1}{\cos^2\varphi}\frac{\partial^2\psi_0}{\partial\lambda\partial\lambda_1} \tag{16}$$

令：

$$B=\frac{1}{a^2\cos\varphi}\frac{\partial}{\partial\varphi}(2\Omega\sin\varphi+a^{-2}\Delta_1\bar{\psi}) \tag{17}$$

又令算子：

$$M=a^{-2}(\frac{\partial}{\partial t}+\frac{\bar{u}}{a\cos\varphi}\frac{\partial}{\partial\lambda})(\frac{\partial^2}{\partial\varphi^2}-\mathrm{tg}\varphi\frac{\partial}{\partial\varphi}+\frac{1}{\cos^2\varphi}\cdot\frac{\partial^2}{\partial\lambda^2}+B\frac{\partial}{\partial\lambda}) \tag{18}$$

因此(14)、(15)、(16)式变为：

$$M(\psi_0)=0 \tag*{(14)$'$}$$

$$M(\psi_1)=F_1 \tag*{(15)$'$}$$

$$M(\psi_2)=F_2 \tag*{(16)$'$}$$

其中：

$$F_1=-a^{-2}(\frac{\partial}{\partial t}+\frac{\bar{u}}{a\cos\varphi}\frac{\partial}{\partial\lambda})(\frac{2}{\cos^2\varphi}\cdot\frac{\partial^2}{\partial\lambda\partial\lambda_1})\psi_0-a^{-2}(\frac{\partial}{\partial\tau}+\frac{\bar{u}}{a\cos\varphi}\frac{\partial}{\partial\lambda_1})(\frac{\partial^2}{\partial\varphi^2}-\mathrm{tg}\varphi\frac{\partial}{\partial\varphi}+\frac{1}{\cos^2\varphi}$$

$$\frac{\partial^2}{\partial\lambda^2})\psi_0-B\frac{\partial\psi_0}{\partial\lambda_1}+\frac{1}{a^4\cos\varphi}\cdot[\frac{\partial\psi_0}{\partial\varphi}\frac{\partial}{\partial\lambda}(\frac{\partial^2}{\partial\varphi^2}-\mathrm{tg}\varphi\frac{\partial}{\partial\varphi}+\frac{1}{\cos^2\varphi}\cdot\frac{\partial^2}{\partial\lambda^2})\psi_0-\frac{\partial\psi_0}{\partial\lambda}\frac{\partial}{\partial\varphi}(\frac{\partial^2}{\partial\varphi^2}-\mathrm{tg}\varphi\frac{\partial}{\partial\varphi}+\frac{1}{\cos^2\varphi}$$

$$\frac{\partial^2}{\partial\lambda^2})\psi_0]-\delta(\frac{\partial^2}{\partial\varphi^2}-\mathrm{tg}\varphi\frac{\partial}{\partial\varphi}+\frac{1}{\cos^2\varphi}\frac{\partial^2}{\partial\lambda^2})\psi_0 \tag{19}$$

$$F_2=-a^{-2}[(\frac{\partial}{\partial t}+\frac{\bar{u}}{a\cos\varphi}\frac{\partial}{\partial\lambda})(\frac{2}{\cos^2\varphi}\frac{\partial^2\psi_1}{\partial\lambda\partial\lambda_1})+(\frac{\partial}{\partial t}+\frac{\bar{u}}{a\cos\varphi}\frac{\partial}{\partial\lambda})(\frac{2}{\cos^2\varphi}\frac{\partial^2\psi_0}{\partial\lambda\partial\lambda_2})+(\frac{\partial}{\partial\tau}+\frac{\bar{u}}{a\cos\varphi}$$

$$\frac{\partial}{\partial\lambda_1})(\frac{\partial^2}{\partial\varphi^2}-\mathrm{tg}\varphi\frac{\partial}{\partial\varphi}+\frac{1}{\cos^2\varphi}\frac{\partial^2}{\partial\lambda^2})\psi_1+(\frac{\partial}{\partial\tau}+\frac{\bar{u}}{a\cos\varphi}\frac{\partial}{\partial\lambda_1})(\frac{2}{\cos^2\varphi}\frac{\partial^2\varphi_0}{\partial\lambda\partial\lambda_1})+(\frac{\partial}{\partial T}+\frac{\bar{u}}{a\cos\varphi}\frac{\partial}{\partial\lambda_2})(\frac{\partial^2}{\partial\varphi^2}-\mathrm{tg}\varphi$$

$$\frac{\partial}{\partial\varphi}+\frac{1}{\cos^2\varphi}\frac{\partial^2}{\partial\lambda^2})\psi_0]-B(\frac{\partial\psi_0}{\partial\lambda_2}+\frac{\partial\psi_1}{\partial\psi_1})-\frac{1}{a^4\cos\varphi}\{\frac{\partial\psi_0}{\partial\lambda}\frac{\partial}{\partial\varphi}(\frac{\partial^2\psi_1}{\partial\varphi^2}-\mathrm{tg}\varphi\frac{\partial\psi_1}{\partial\varphi}+\frac{1}{\cos^2\varphi}\frac{\partial^2\psi_1}{\partial\lambda^2}+\frac{2}{\cos^2\varphi}\frac{\partial^2\psi_0}{\partial\lambda\partial\lambda_1})+$$

$$(\frac{\partial\psi_1}{\partial\lambda}+\frac{\partial\psi_0}{\partial\lambda_1})\frac{\partial}{\partial\varphi}(\frac{\partial^2}{\partial\varphi^2}-\mathrm{tg}\varphi\frac{\partial}{\partial\varphi}+\frac{1}{\cos^2\varphi}\frac{\partial^2}{\partial\lambda^2})\psi_0-[\frac{\partial\psi_0}{\partial\varphi}\frac{\partial}{\partial\lambda}(\frac{\partial^2}{\partial\varphi^2}-\mathrm{tg}\varphi\frac{\partial}{\partial\varphi}+\frac{1}{\cos^2\varphi}\cdot\frac{\partial^2}{\partial\lambda^2})\psi_1+\frac{\partial\psi_0}{\partial\varphi}\frac{2}{\cos^2\varphi}$$

$$\frac{\partial^3\psi_0}{\partial\lambda_1\partial\lambda^2}+\frac{\partial\psi_0}{\partial\varphi}\frac{\partial}{\partial\lambda_1}(\frac{\partial^2}{\partial\varphi^2}-\mathrm{tg}\varphi\frac{\partial}{\partial\varphi}+\frac{1}{\cos^2\varphi}\cdot\frac{\partial^2}{\partial\lambda^2})\psi_0+\frac{\partial\psi_1}{\partial\varphi}\frac{\partial}{\partial\lambda}(\frac{\partial^2}{\partial\varphi^2}-\mathrm{tg}\varphi\frac{\partial}{\partial\varphi}+\frac{1}{\cos^2\varphi}\cdot\frac{\partial^2}{\partial\lambda^2})\psi_0+\frac{\partial\psi_0}{\partial\varphi}\cdot$$

$$(\frac{2}{\cos^2\varphi}\cdot\frac{\partial^3\psi_0}{\partial\lambda_1\partial\lambda^2})]\}-\delta a^{-2}(\frac{\partial^2}{\partial\varphi^2}-\mathrm{tg}\varphi\frac{\partial}{\partial\varphi}+\frac{1}{\cos^2\varphi}\frac{\partial^2}{\partial\lambda^2})\psi_1-\delta\frac{a^{-2}}{\cos^2\varphi}\frac{\partial^2\psi_0}{\partial\lambda\partial\lambda_1} \tag{20}$$

在获得球面扰动波包的演变方程之前，我们必须分离出快变量和慢变量对流函数的影响。一般认为扰动的波包仅是缓变量的函数。为了使多重尺度分析方法所得到的解有物理意义，Maslowe[5]在讨论切变层中弱非线性作用的稳定性问题时，给出了一种分离快

变量和慢变量对函数作用的方法,据此,我们设:

$$\psi = \varepsilon(\phi_0 e^{ik(\lambda-ct)} + *) + \varepsilon^2(\phi_1 e^{2ik(\lambda-ct)} + *) + \varepsilon^3\psi_2 + \cdots \tag{21}$$

这里的"*"代表其前面相应项的复数共轭,式中的 c 相当于波的传播速度,k 亦类似于波数,但它们都与通常的平面坐标意义下的量有所不同。又设:

$$\psi_0 = \phi_0 e^{ik(\lambda-ct)} + * = \Phi_0(\varphi)A(\tau,T;\lambda_1,\lambda_2)e^{ik(\lambda-ct)} + * \tag{22}$$

$$\psi_1 = \phi_1 e^{2ik(\lambda-ct)} + * = \Phi_1(\varphi)A^2(\tau、T;\lambda_1,\lambda_2)e^{2ik(\lambda-ct)} + * \tag{23}$$

(22)、(23)式也是根据 Maslowe 的方法[5]而设的,在文献[5]中,Maslowe 得到了一个与非线性的 Schrödinger 方程类似的波幅方程。(23)、(24)式中的 A 为扰动的波包络,它只是缓变量的函数。将(21)—(23)式代入(14)′—(20)式,并根据多重尺度分析方法的原则:欲使所求之解收敛(具有物理意义),则必须消除方程中的长期项(实际上,设 ψ_1 解的形式时也是基于这一思想考虑的),可得以下一系列方程:

$$\cos^2\varphi\frac{d^2\Phi_0}{d\varphi^2} - \sin\varphi\cos\varphi\frac{d\Phi_0}{d\varphi} + \left(\frac{a^3B\cos^3\varphi}{\bar{u} - ca\cos\varphi} - k^2\right)\Phi_0 = 0 \tag{24}$$

$$2a(\bar{u} - ca\cos\varphi)\left(\frac{d^2\Phi_1}{d\varphi^2} - \mathrm{tg}\varphi\frac{d\Phi_1}{d\varphi} - \frac{4k^2}{\cos^2\varphi}\Phi_1\right) = \Phi_0\frac{d}{d\varphi}\left(\frac{aB\cos\varphi}{\bar{u} - ca\cos\varphi}\right) \tag{25}$$

以及

$$\frac{2k^2\Phi_0}{a^2\cos^2\varphi}\left(\frac{\bar{u}}{a\cos\varphi} - c\right)\frac{\partial A}{\partial\lambda_2} - ik\frac{2\Phi_0}{a^2\cos^2\varphi}\left(\frac{\partial}{\partial\tau} + \frac{\bar{u}}{a\cos\varphi}\cdot\frac{\partial}{\partial\lambda_1}\right)\frac{\partial A}{\partial\lambda_1} - \frac{1}{a^2}\left(\frac{d^2\Phi_0}{d\varphi^2} - \frac{k^2}{\cos^2\varphi}\Phi_0 - \mathrm{tg}\varphi\frac{d\Phi_0}{d\varphi}\right)$$
$$\left(\frac{\partial}{\partial T} + \frac{\bar{u}}{a\cos\varphi}\cdot\frac{\partial}{\partial\lambda_2}\right)A + B\Phi_0\frac{\partial A}{\partial\lambda_2} + \frac{ik|A|^2A}{a^4\cos\varphi}\Big[\Phi_0^*\frac{d}{d\varphi}\left(\frac{d^2\Phi_1}{d\varphi^2} - \mathrm{tg}\frac{d\Phi_1}{d\varphi} - \frac{4k^2}{\cos^2\varphi}\Phi_1\right) + 2\Phi_1\frac{d}{d\varphi}\Big(\frac{d^2\Phi_0^*}{d\varphi^2} - \mathrm{tg}\varphi$$
$$\frac{d\Phi_0^*}{d\varphi} - \frac{k^2\Phi_0^*}{\cos^2\varphi}\Big) + 2\frac{d\Phi_0^*}{d\varphi}\left(\frac{d^2\Phi_1}{d\varphi^2} - \mathrm{tg}\varphi\frac{d\Phi_1}{d\varphi} - \frac{4k^2}{\cos^2\varphi}\Phi_1 - \frac{d\Phi_1}{d\varphi}\left(\frac{d^2\Phi_0^*}{d\varphi^2} - \mathrm{tg}\varphi\frac{d\Phi_0^*}{d\varphi} - \frac{k^2}{\cos^2\varphi}\Phi_0^*\right)\right)\Big] - \delta$$
$$\frac{\Phi_0 a^{-2}}{\cos^2\varphi}\frac{\partial^2 A}{\partial\lambda\partial\lambda_1} = 0 \tag{26}$$

尽管我们一再强调扰动是小振幅的(即弱非线性问题),但这只意味着振幅 A 比起背景场运动的幅度来说是较小的,而不是 A 本身,也就是说,扰动具有一定的宽度。况且在较高纬度地区,Rossby 参数 β 随纬度的增高而增长很快。鉴于上述原因,我们必须考虑扰动的球面效应。很显然,当 Φ_0 和 Φ_1 的边界条件给定时,我们便可以计算(24)式中各项的系数。设球面上的小扰动只发生在通道$[\varphi_1、\varphi_2]$中,故有 $\Phi_0(\varphi_1)=\Phi_0(\varphi_2)=0$,这样的边界条件与(24)式便构成了 Φ_0 的本征值问题,Φ_0 可求。同样,令 $\Phi_1(\varphi_1)=\Phi_1(\varphi_2)=0$ 也与(25)式构成了关于 Φ_1 的本征值问题,Φ_1 可求。根据以上的讨论可知:(26)式中只有波包的缓变波幅这个变量,即 A 为调制波的波幅。为了使方程(26)式的物理意义更加清楚,用 $a^2\cos^2\varphi\frac{d^2\Phi_0}{d\varphi^2}$ 乘方程(26)式的两边,然后再在$[\varphi_1、\varphi_2]$上积分,并注意 Φ_0、Φ_1 的边界条件及波幅 A 与快变量 φ 无关,则可以得到波包的演变方程:

$$\frac{\partial A}{\partial T} + c_g\frac{\partial A}{\partial\lambda_2} + ikl\frac{\partial^2 A}{\partial\lambda_1^2} + h\frac{\partial^2 A}{\partial\lambda\partial\lambda_1} + ikm\varepsilon^2|A|^2A = 0 \tag{27}$$

式中:

$$h = \delta\cdot\int_{\varphi_1}^{\varphi_2}\left(\frac{d\Phi_0}{d\varphi}\right)^2 d\varphi\Big/\int_{\varphi_1}^{\varphi_2}\frac{a^3B\cos^3\varphi}{\bar{u} - ca\cos\varphi}\Phi_0\frac{d\Phi_0}{d\varphi}d\varphi \tag{28}$$

h是由大气粘性力造成的，$h\frac{\partial^2 A}{\partial\lambda_1\partial\lambda}$项相当于一种耗散作用。若无粘性力作用，在展开过程中用其它的小参数代替 $\varepsilon=\frac{K}{\delta}$，则最后的波包方程中也没有这项，方程形式为标准的非线性 Schrödinger 方程，即波包呈孤立波状态[6]。也就是说，由于粘性力的存在，使波包的耗散性加强了，不利于孤立波的形成与维持，从另一个角度证实了在摩擦层中难以看到长时间维持的天气系统。(27)式的其它系数及意义亦可参见文献[6]。从前面的推导过程中可以看到，由于认为粘性力的作用比扰动作用小一个量级，它导致波包是缓慢地耗散的；如果粘性力作用加大，则波包的耗散性加强，波包成为快变；如果粘性力小一个量级，则波包在缓变尺度上就成为孤立波了。我们以前也曾将非线性 Schrödinger 方程用于讨论气象中的中、小尺度系统演变与发展，认为斜压性(基本风场的垂直切变)很重要，没有它，孤立波难以形成[7]。由此可以发现，风场的水平经向切变对孤立波的形成起很重要的作用。

三、讨　论

通过考虑球面扰动弱非线性相互作用的因素及粘性力的作用，获得了球面扰动的包络演变方程。该演变方程粘性力为零(或小一个量级)时，它表现为非线性 Schrödinger 方程。而此方程的成立则依赖于运动的球面性及基本风场的经向切变，粘性力的作用加强了扰动能量的耗散作用，不利于孤立波的形成。

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THE CHARACTERS OF VISCOUS BAROTROPIC ATMOSPHERE ON SPHERE

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Abstract

The characters of nonlinear disturbance of viscous barotropic atmosphere on sphere is discussed. It shows that the envelop of disturbance is controlled by a simple equation because of the reactions of weak nonlinear interaction between waves. The simple equation changes into nonlinear Schrödinger equation when viscous effect is died away. The result also shows that two factors, one is the effect of curvature of the sphere and the other is the shear of the basic wind field, are very important for the formation of the equation. The paper is also pointed out that the viscous effect may act as dispersion to enhance the energy of the envelop and it may be unfavourable for the formation of this statistical solitary wave (Schrödinger wave).