

含大地形的准地转正压模式的孤立波解*

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提 要

该文由修正过的含大地形的准地转正压模式方程出发, 考虑青藏高原大地形的实际情况, 忽略其东西向地形坡度, 再利用约化摄动方法, 求其孤立波解, 并得到结论: 当基本气流无切变时, 地形是产生 Rossby 孤立波的必要因子。

关键词: 准地转正压模式 约化摄动法 孤立波

引 言

青藏高原占我国陆地面积约四分之一, 平均海拔在 4 km 以上, 是世界上最高、最陡峭的大地形. 国内许多研究证明^[1~7]: 青藏高原不仅影响我国和东亚的天气气候, 而且对全球性的大气环流均有影响. 吕克利^[8,9]曾讨论过大地形对正压 Rossby 孤立波的影响. 在文献^[10]中, 作者曾推导出含青藏高原大地形、摩擦和加热的准地转正压模式方程. 本文以这个方程为出发点, 不考虑摩擦和加热, 利用约化摄动法^[11], 求出其孤立波解^[12].

1 基本方程

不考虑摩擦和加热, 含大地形的准地转正压模式方程为:

$$\begin{aligned} & \left(\frac{\partial}{\partial t} + \frac{\partial \Psi}{\partial x} \frac{\partial}{\partial y} - \frac{\partial \Psi}{\partial y} \frac{\partial}{\partial x} \right) (f + \nabla_h^2 \Psi) \\ &= \frac{f_0^2}{C_0^2 - f_0 \Psi_s} \left(\frac{\partial}{\partial t} + \frac{\partial \Psi}{\partial x} \frac{\partial}{\partial y} - \frac{\partial \Psi}{\partial y} \frac{\partial}{\partial x} \right) (\Psi - \Psi_s) \end{aligned} \quad (1)$$

其中 f 为 Coriolis 参数, f_0 是其特征值, 为一常数, $C_0 = \sqrt{gH}$ (H 为大气标高, g 为重力加速度), Ψ 为准地转流函数, $\Psi_s = \frac{gh_s}{f_0}$ (h_s 为地形高度) 为地形流函数.

$$\nabla_h^2 \equiv \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} \quad (2)$$

令:

* 本文得到非线性斑图动力学和《国家重点基础研究发展规划》首批启动的《我国重大气候灾害的机理和预测理论》研究项目的资助. 1997-09-29 收到, 1998-07-13 收到修改稿.

$$\Psi = \bar{\Psi}(y) + \Psi', \quad \bar{u} = -\frac{\partial \bar{\Psi}}{\partial y} \quad (3)$$

将式(3)代入式(1)得:

$$\begin{aligned} & \left(\frac{\partial}{\partial t} + \bar{u} \frac{\partial}{\partial x}\right) \nabla_h^2 \Psi' + \left(\frac{\partial \Psi'}{\partial x} \frac{\partial}{\partial y} - \frac{\partial \Psi'}{\partial y} \frac{\partial}{\partial x}\right) \left(f + \frac{\partial^2 \bar{\Psi}}{\partial y^2} + \nabla_h^2 \Psi'\right) \\ &= \frac{f_0^2}{C_0^2 - f_0 \Psi_s} \left[\left(\frac{\partial}{\partial t} + \bar{u} \frac{\partial}{\partial x}\right) (\Psi' - \Psi_s) + \left(\frac{\partial \Psi'}{\partial x} \frac{\partial}{\partial y} - \frac{\partial \Psi'}{\partial y} \frac{\partial}{\partial x}\right) (\bar{\Psi} + \Psi' - \Psi_s) \right] \quad (4) \end{aligned}$$

整理得:

$$\begin{aligned} & \left(\frac{\partial}{\partial t} + \bar{u} \frac{\partial}{\partial x}\right) \nabla_h^2 \Psi' - \frac{f_0^2}{C_0^2 - f_0 \Psi_s} \frac{\partial \Psi'}{\partial t} + \left(\beta_0 - \frac{\partial^2 \bar{u}}{\partial y^2}\right) \frac{\partial \Psi'}{\partial x} + J(\Psi', \nabla_h^2 \Psi') \\ &= \frac{f_0^2}{C_0^2 - f_0 \Psi_s} \left[-\bar{u} \frac{\partial \Psi_s}{\partial x} - J(\Psi', \Psi_s) \right] \quad (5) \end{aligned}$$

式中: β_0 是 Rossby 参数, 为一常数, J 为 Jacobi 算子, 即:

$$J(A, B) = \frac{\partial A}{\partial x} \frac{\partial B}{\partial y} - \frac{\partial A}{\partial y} \frac{\partial B}{\partial x} \quad (6)$$

考虑青藏高原的地形特征, 其东西向地形坡度可忽略不计, 即: $\frac{\partial \Psi_s}{\partial x} = 0$, 则式(5)可化为:

$$\begin{aligned} & \left(\frac{\partial}{\partial t} + \bar{u} \frac{\partial}{\partial x}\right) \nabla_h^2 \Psi' - \frac{f_0^2}{C_0^2 - f_0 \Psi_s} \frac{\partial \Psi'}{\partial t} + \left(\beta_0 - \frac{\partial^2 \bar{u}}{\partial y^2}\right) \frac{\partial \Psi'}{\partial x} + J(\Psi', \nabla_h^2 \Psi') \\ &= \frac{f_0^2}{C_0^2 - f_0 \Psi_s} \frac{\partial \Psi'}{\partial x} \frac{\partial \Psi_s}{\partial y} \quad (7) \end{aligned}$$

下面我们就利用约化摄动法将式(5)化为 KdV 方程, 再求其孤立波解.

2 KdV 方程的导出

对于复杂的非线性方程, 通过 Gardenr-Morikawa 变换(简称 G-M 变换)和摄动法化为简单的能准确求解的非线性方程, 这种方法称为约化摄动法.

(1) G-M 变换

令:

$$\xi = \varepsilon^{\frac{1}{2}}(x - ct), \quad \tau = \varepsilon^{\frac{3}{2}}t, \quad y = y \quad (8)$$

其中: ε 为小参数, c 为坐标 x 的平移速度. 则:

$$\begin{cases} \frac{\partial}{\partial t} = -\frac{\partial}{\partial \xi} c \varepsilon^{\frac{1}{2}} + \frac{\partial}{\partial \tau} \varepsilon^{\frac{3}{2}} \\ \frac{\partial}{\partial x} = \frac{\partial}{\partial \xi} \varepsilon^{\frac{1}{2}} \\ \frac{\partial}{\partial y} = \frac{\partial}{\partial y} \end{cases} \quad (9)$$

将式(9)代入式(7)有:

$$\left[\varepsilon \frac{\partial}{\partial \tau} + (\bar{u} - c) \frac{\partial}{\partial \xi} \right] \left(\varepsilon \frac{\partial^2 \Psi'}{\partial \xi^2} + \frac{\partial^2 \Psi'}{\partial y^2} \right) - \frac{f_0^2}{C_0^2 - f_0 \Psi_s} \left(\varepsilon \frac{\partial \Psi'}{\partial \tau} - c \frac{\partial \Psi'}{\partial y} \right) +$$

$$\begin{aligned} & (\beta_0 - \frac{\partial \bar{u}}{\partial y^2}) \frac{\partial \Psi'}{\partial \xi} + \frac{\partial \Psi'}{\partial \xi} \frac{\partial}{\partial y} (\epsilon \frac{\partial^2 \Psi'}{\partial \xi^2} + \frac{\partial^2 \Psi'}{\partial y^2}) - \\ & \frac{\partial \Psi'}{\partial y} \frac{\partial}{\partial \xi} (\epsilon \frac{\partial^2 \Psi'}{\partial \xi^2} + \frac{\partial^2 \Psi'}{\partial y^2}) = \frac{f_0^2}{C_0^2 - f_0 \Psi_s} \frac{\partial \Psi_s}{\partial y} \frac{\partial \Psi'}{\partial \xi} \end{aligned} \quad (10)$$

整理得:

$$\begin{aligned} & (\bar{u} - c) \frac{\partial}{\partial \xi} \frac{\partial^2 \Psi'}{\partial y^2} + [\frac{f_0^2}{C_0^2 - f_0 \Psi_s} (c + \frac{\partial \Psi_s}{\partial y}) + \beta_0 - \frac{\partial \bar{u}}{\partial y^2}] \frac{\partial \Psi'}{\partial \xi} + \\ & \frac{\partial \Psi'}{\partial \xi} \frac{\partial^2 \Psi'}{\partial y^3} - \frac{\partial \Psi'}{\partial y} \frac{\partial^3 \Psi'}{\partial \xi \partial y^2} + \epsilon [\frac{\partial}{\partial \tau} \frac{\partial^2 \Psi'}{\partial y^2} + (u - c) \frac{\partial^3 \Psi'}{\partial \xi^3} - \\ & \frac{f_0^2}{C_0^2 - f_0 \Psi_s} \frac{\partial \Psi'}{\partial \tau} + \frac{\partial \Psi'}{\partial \xi} \frac{\partial^3 \Psi'}{\partial y^2 \partial \xi} - \frac{\partial \Psi'}{\partial y} \frac{\partial^3 \Psi'}{\partial \xi^3}] + \epsilon^2 \frac{\partial}{\partial \tau} \frac{\partial^2 \Psi'}{\partial \xi^2} = 0 \end{aligned} \quad (11)$$

给定齐次边界条件:

$$\Psi' |_{y=y_1} = 0, \Psi' |_{y=y_2} = 0 \quad (12)$$

(2) 摄动

令:

$$\Psi' = \epsilon \Psi_1 + \epsilon^2 \Psi_2 + \dots \quad (13)$$

代入方程(11), 则求得它的一级近似方程为:

$$(\bar{u} - c) \frac{\partial}{\partial \xi} \frac{\partial^2 \Psi_1}{\partial y^2} + [\frac{f_0^2}{C_0^2 - f_0 \Psi_s} (c + \frac{\partial \Psi_s}{\partial y}) + \beta_0 - \frac{\partial \bar{u}}{\partial y^2}] \frac{\partial \Psi_1}{\partial \xi} = 0 \quad (14)$$

它的二级近似方程为:

$$\begin{aligned} & (\bar{u} - c) \frac{\partial}{\partial \xi} \frac{\partial^2 \Psi_2}{\partial y^2} + [\frac{f_0^2}{C_0^2 - f_0 \Psi_s} (c + \frac{\partial \Psi_s}{\partial y}) + \beta_0 - \frac{\partial \bar{u}}{\partial y^2}] \frac{\partial \Psi_2}{\partial \xi} = - \frac{\partial}{\partial \tau} \frac{\partial^2 \Psi_1}{\partial y^2} + \\ & \frac{f_0^2}{C_0^2 - f_0 \Psi_s} \frac{\partial \Psi_1}{\partial \tau} - (u - c) \frac{\partial^3 \Psi_1}{\partial \xi^3} - \frac{\partial \Psi_1}{\partial \xi} \frac{\partial^3 \Psi_1}{\partial y^3} - \frac{\partial \Psi_1}{\partial y} \frac{\partial^3 \Psi_1}{\partial \xi \partial y^2} \end{aligned} \quad (15)$$

式(14)中 Ψ_1 对 (ξ, τ) 和 y 是可分离的, 令:

$$\Psi_1 = A(\xi, \tau)G(y) \quad (16)$$

将式(16)代入式(14), 整理后可得:

$$\frac{\partial A}{\partial \xi} \{ (\bar{u} - c) \frac{\partial^2 G}{\partial y^2} + [\frac{f_0^2}{C_0^2 - f_0 \Psi_s} (c + \frac{\partial \Psi_s}{\partial y}) + \beta_0 - \frac{\partial \bar{u}}{\partial y^2}] G \} = 0 \quad (17)$$

$\frac{\partial A}{\partial \xi} \neq 0$, 因此得:

$$(\bar{u} - c) \frac{\partial^2 G}{\partial y^2} + [\frac{f_0^2}{C_0^2 - f_0 \Psi_s} (c + \frac{\partial \Psi_s}{\partial y}) + \beta_0 - \frac{\partial \bar{u}}{\partial y^2}] G = 0 \quad (18)$$

令:

$$Q(y) = \frac{\frac{f_0^2}{C_0^2 - f_0 \Psi_s} (c + \frac{\partial \Psi_s}{\partial y}) + \beta_0 - \frac{\partial \bar{u}}{\partial y^2}}{\bar{u} - c} \quad (19)$$

则由式(18)和边界条件式(12)得到了关于 $G(y)$ 的特征值问题:

$$\begin{cases} \frac{\partial^2 G}{\partial y^2} + Q(y)G = 0 \\ G|_{y=y_1} = 0, G|_{y=y_2} = 0 \end{cases} \quad (20)$$

如果 $Q(y) > 0$, 则 Ψ_1 在 y 方向上表现为波动. 例如:

(i) 不考虑地形, 且 \bar{u} 为常数, 则:

$$Q = \frac{\lambda_0^2 c + \beta_0}{\bar{u} - c} = l^2 \quad (21)$$

式中 $\lambda_0 = \frac{f_0}{C_0}$, λ_0^{-1} 为正压 Rossby 变形半径. 则由上式可得:

$$c = \frac{-\beta_0 + l^2 \bar{u}}{\lambda_0^2 + l^2} \quad (22)$$

这是 x 方向上的波数 $k \rightarrow 0$ 时的 Rossby 波速.

(ii) 考虑地形, 但 $\frac{\partial \Psi_s}{\partial y}$ 为常数, 且 \bar{u} 为常数, 令: $\beta_s = \lambda_0^2 \frac{\partial \Psi_s}{\partial y}$, β_s 为表示南北向地形坡度的参数, 可称为地形 Rossby 参数, 则:

$$Q \approx \frac{\lambda_0^2 c + \beta_s + \beta_0}{\bar{u} - c} = l^2 \quad (23)$$

所以得:

$$c = \frac{-(\beta_0 + \beta_s) + l^2 \bar{u}}{\lambda_0^2 + l^2} \quad (24)$$

这是 $k \rightarrow 0$ 时含地形南北坡度时的 Rossby 波速.

考虑二级近似方程(15), 将式(16)和式(19)代入式(15), 得:

$$\begin{aligned} (\bar{u} - c) \frac{\partial}{\partial \xi} \left[\frac{\partial^2 \Psi_2}{\partial y^2} + Q(y) \Psi_2 \right] = & - \left(\frac{\partial A}{\partial \xi} \frac{\partial G}{\partial y^2} - \frac{f_0^2}{C_0^2 - f_0 \Psi_s} G \frac{\partial A}{\partial \xi} \right) - \\ & (\bar{u} - c) \frac{\partial^3 A}{\partial \xi^3} G - \left(G \frac{\partial A}{\partial \xi} \frac{\partial^2 G}{\partial \xi^3} - A \frac{\partial G}{\partial y} \frac{\partial A}{\partial \xi} \frac{\partial^2 G}{\partial y^2} \right) \end{aligned} \quad (25)$$

将上式两边同除 $\bar{u} - c$, 注意: $\frac{\partial^2 G}{\partial y^2} = -Q(y)G$, 得:

$$\begin{aligned} \frac{\partial}{\partial \xi} \left[\frac{\partial^2 \Psi_2}{\partial y^2} + Q(y) \Psi_2 \right] = & \frac{Q(y) + \frac{f_0^2}{C_0^2 - f_0 \Psi_s}}{\bar{u} - c} G \frac{\partial A}{\partial \xi} - G \frac{\partial^3 A}{\partial \xi^3} - \\ & \frac{1}{\bar{u} - c} \left(G \frac{\partial^2 G}{\partial y^3} - \frac{\partial G}{\partial y} \frac{\partial^2 G}{\partial y^2} \right) A \frac{\partial A}{\partial \xi} \end{aligned} \quad (26)$$

上式两端乘 G 后对 y 积分, 利用式(20)有:

$$\left\{ \begin{aligned} \text{左边} &= \frac{\partial}{\partial \xi} \int_{y_1}^{y_2} G \left(\frac{\partial^2 \Psi_2}{\partial y^2} + Q \Psi_2 \right) dy = \\ & \frac{\partial}{\partial \xi} \left[\int_{y_1}^{y_2} \frac{\partial}{\partial y} G \left(G \frac{\partial \Psi_2}{\partial y} - \Psi_2 \frac{\partial G}{\partial y} \right) dy + \int_{y_1}^{y_2} \Psi_2 \left(\frac{\partial^2 G}{\partial y^2} + QG \right) dy \right] = 0 \\ \text{右边} &= \int_{y_1}^{y_2} \left\{ \frac{Q(y) + \frac{f_0^2}{C_0^2 - f_0 \Psi_s}}{\bar{u} - c} G^2 \frac{\partial A}{\partial \xi} - G^2 \frac{\partial^3 A}{\partial \xi^3} - \right. \\ & \left. \frac{G}{\bar{u} - c} \left(G \frac{\partial^2 G}{\partial y^3} - \frac{\partial G}{\partial y} \frac{\partial^2 G}{\partial y^2} \right) A \frac{\partial A}{\partial \xi} \right\} dy \\ &= I_1 \frac{\partial A}{\partial \xi} + I_2 A \frac{\partial A}{\partial \xi} + I_3 \frac{\partial^3 A}{\partial \xi^3} \end{aligned} \right. \quad (27)$$

其中:

$$\begin{cases} I_1 = \int_{y_1}^{y_2} \frac{Q(y) + \frac{f_0^2}{C_0^2 - f_0 \Psi_s}}{\bar{u} - c} G^2 dy \\ I_2 = \int_{y_1}^{y_2} -\frac{G}{\bar{u} - c} \left(G \frac{\partial^3 G}{\partial y^3} - \frac{\partial G}{\partial y} \frac{\partial^2 G}{\partial y^2} \right) dy \\ I_3 = \int_{y_1}^{y_2} -G^2 dy \end{cases} \quad (28)$$

这样式(26)可化为:

$$\frac{\partial A}{\partial \tau} + \alpha A \frac{\partial A}{\partial \xi} + \gamma \frac{\partial^3 A}{\partial \xi^3} = 0 \quad (29)$$

其中:

$$\begin{cases} \alpha = \frac{I_2}{I_1} = \frac{\int_{y_1}^{y_2} -\frac{G}{\bar{u} - c} \left(G \frac{\partial^3 G}{\partial y^3} - \frac{\partial G}{\partial y} \frac{\partial^2 G}{\partial y^2} \right) dy}{\int_{y_1}^{y_2} \frac{Q(y) + \frac{f_0^2}{C_0^2 - f_0 \Psi_s}}{\bar{u} - c} G^2 dy} \\ \gamma = \frac{I_3}{I_1} = \frac{\int_{y_1}^{y_2} -G^2 dy}{\int_{y_1}^{y_2} \frac{Q(y) + \frac{f_0^2}{C_0^2 - f_0 \Psi_s}}{\bar{u} - c} G^2 dy} \end{cases} \quad (30)$$

式(29)即是式(1)通过约化摄动法所得到的 KdV 方程。

3 KdV 方程的求解

下面用行波法来求解 KdV 方程(29)的椭圆余弦波解和孤立波解,令:

$$A = A(\eta), \quad \eta = \xi - c_1 \tau = \varepsilon^{\frac{1}{2}} [x - (c + \varepsilon c_1)t] \quad (31)$$

其中 $\xi - c_1 \tau$ 可视为波的等位相线,它的移动速度即为 c_1 ,则式(29)化为:

$$-c_1 \frac{dA}{d\eta} + \alpha A \frac{dA}{d\eta} + \gamma \frac{d^3 A}{d\eta^3} = 0 \quad (32)$$

式(32)对 η 积分一次,得:

$$-c_1 A + \alpha A^2 + \gamma \frac{d^2 A}{d\eta^2} = B_0 \quad (33)$$

其中 B_0 为积分常数,式(33)两边同乘 $\frac{dA}{d\eta}$ 后再对 η 积分,整理后得:

$$\left(\frac{dA}{d\eta} \right)^2 = -\frac{\alpha}{3\gamma} (A^3 - \frac{3c_1}{\alpha} A^2 - \frac{6B_0}{\alpha} A - \frac{6B_1}{\alpha}) \quad (34)$$

其中 B_1 为积分常数,令:

$$P(A) = A^3 - \frac{3c_1}{\alpha} A^2 - \frac{6B_0}{\alpha} A - \frac{6B_1}{\alpha}$$

$$= (A - A_1)(A - A_2)(A - A_3) \quad (35)$$

A_1, A_2, A_3 是 $P(A)$ 的 3 个零点, 且 $A_1 > A_2 > A_3$. 由式(35)可得:

$$\begin{cases} c_1 = \frac{\alpha}{3}(A_1 + A_2 + A_3) \\ B_0 = -\frac{\alpha}{6}(A_1A_2 + A_2A_3 + A_1A_3) \\ B_1 = \frac{\alpha}{6}A_1A_2A_3 \end{cases} \quad (36)$$

下面分两种情况来讨论:

(1) $\alpha\gamma > 0$

$$\begin{aligned} A(\xi, \tau) &= A(\eta) = A_2 + (A_1 - A_2)\text{cn}^2 \sqrt{\frac{\alpha}{12\gamma}(A_1 - A_3)}\eta \\ &= A_2 + (A_1 - A_2)\text{cn}^2 \sqrt{\frac{\alpha}{12\gamma}(A_1 - A_3)}\varepsilon^{\frac{1}{2}}[x - (c + \varepsilon c_1)t] \end{aligned} \quad (37)$$

其中模数为 k ,

$$k^2 = \frac{A_1 - A_2}{A_1 - A_3} \quad (38)$$

若 $A_2 \rightarrow A_3, k \rightarrow 1$, 则式(37)可化为:

$$\begin{aligned} A(\xi, \tau) &= A(\eta) = A_2 + (A_1 - A_2)\text{sech}^2 \sqrt{\frac{\alpha}{12\gamma}(A_1 - A_3)}\eta \\ &= A_2 + (A_1 - A_2)\text{sech}^2 \sqrt{\frac{\alpha}{12\gamma}(A_1 - A_3)}\varepsilon^{\frac{1}{2}}[x - (c + \varepsilon c_1)t] \end{aligned} \quad (39)$$

其中:

$$\begin{cases} c_1 = \frac{\alpha}{3}(A_1 + 2A_2) = \alpha[A_2 + \frac{1}{3}(A_1 - A_2)] = \alpha(A_2 + \frac{1}{3}a) \\ a = A_1 - A_2 \end{cases} \quad (40)$$

(2) $\alpha\gamma < 0$

$$\begin{aligned} A(\xi, \tau) &= A(\eta) = A_2 - (A_2 - A_3)\text{cn}^2 \sqrt{\left| \frac{\alpha}{12\gamma} \right| (A_1 - A_3)}\eta \\ &= A_2 - (A_2 - A_3)\text{cn}^2 \sqrt{\left| \frac{\alpha}{12\gamma} \right| (A_1 - A_3)}\varepsilon^{\frac{1}{2}}[x - (c + \varepsilon c_1)t] \end{aligned} \quad (41)$$

其中:

$$k^2 = \frac{A_2 - A_3}{A_1 - A_3} \quad (42)$$

若 $A_1 \rightarrow A_2, k \rightarrow 1$, 则式(41)可化为:

$$\begin{aligned} A(\xi, \tau) &= A(\eta) = A_2 - (A_2 - A_3)\text{sech}^2 \sqrt{\left| \frac{\alpha}{12\gamma} \right| (A_1 - A_3)}\eta \\ &= A_2 - (A_2 - A_3)\text{sech}^2 \sqrt{\left| \frac{\alpha}{12\gamma} \right| (A_1 - A_3)}\varepsilon^{\frac{1}{2}}[x - (c + \varepsilon c_1)t] \end{aligned} \quad (43)$$

$$\begin{cases} c_1 = \frac{\alpha}{3}(2A_2 + A_3) = \alpha[A_2 - \frac{1}{3}(A_2 - A_3)] = \alpha(A_2 - \frac{1}{3}a) \\ a = A_2 - A_3 \end{cases} \quad (44)$$

式(37)和式(41)是 KdV 方程(29)的椭圆余弦波解,式(39)和式(43)是其孤立波解,其中式(40)和式(44)中的 a 分别为两种情况下的椭圆余弦波和孤立波的振幅,它们在 x 方向的移速为 $c + \epsilon c_1$.

4 结 论

(1)式(24)为考虑青藏高原大地形南北坡度时且在 x 方向为长波近似下的 Rossby 波速,当忽略地形南北坡度参数时,式(24)就化为式(22),式(22)即通常的且在 x 方向为长波近似下的 Rossby 波速.比较式(22)和式(24)可知,地形的影响使在地形南坡 Rossby 波有向东传播的趋势,而在北坡加速其向西的传播.

(2)含大地形的准地转正压模式方程的椭圆余弦波解为式(37)和式(41),孤立波解为式(39)和式(43).

(3)当 \bar{u} 为常数时,如不考虑地形,则 Q 与 y 无关,由式(20)可知: $G \frac{\partial^2 G}{\partial y^3} - \frac{\partial G}{\partial y} \frac{\partial^2 G}{\partial y^2} = 0$,则式(30)中 $\alpha = 0$,KdV 方程无孤立波解.如考虑地形,即使 \bar{u} 为常数,但 $\alpha \neq 0$,仍有孤立波产生.

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SOLITARY WAVES OF THE BAROTROPIC QUASI-GEOSTROPHIC MODEL WITH LARGE-SCALE OROGRAPHY

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Abstract

Starting from a modified barotropic quasi-geostrophic model equation, considering the actual situation of the large-orography of the Qinghai-Xizang Plateau, neglecting its east-west slope, the solitary waves are obtained using reductive perturbation method. The results show that the orography is an essential factor to excite Rossby solitary waves in basic flow without shear.

Key words: Barotropic quasi-geostrophic model Reductive perturbation method Solitary wave

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